PHYSICAL REVIEW E, VOLUME 64, 035402(R)

Oscillations in a chain of rod-shaped colloidal particles in a plasma

S. V. Vladimirov* and E. N. Tsoy
Department of Theoretical Physics, School of Physics, The University of Sydney, New South Wales 2006, Australia
(Received 23 May 2001; published 29 August 2001)

Oscillations in the one-dimensional chain consisting of rotatorlike particles levitating in a plasma are studied. General equations of motion for such a chain are derived. It is demonstrated that new oscillation modes associated with the rotational degrees of freedom appear for such a configuration. The dispersion characteristics of the modes are analyzed. Collective oscillations and equilibrium in lattices composed of cylindrical particles in a plasma are discussed.

DOI: 10.1103/PhysRevE.64.035402 PACS number(s): 52.35.Ra, 52.35.Kt, 52.25.Vy

Experiments [1–4] involving colloidal charged particles levitating in the plasma have attracted serious attention. Formation of colloidal crystals and phase transitions in these systems [4] are important fundamental questions. At present, most of the cases studied experimentally and theoretically correspond to spherical dust grains.

However, there are recent experimental observations of the formation of colloidal structures composed of elongated (cylindrical) particles [5,6] levitating in the sheath region of a gas discharge plasma. The experiments demonstrate that there are various arrangements of such grains, levitating horizontally (i.e., oriented parallel to the lower electrode and perpendicular to the gravity force) and vertically (i.e., oriented perpendicular to the lower electrode and parallel to the gravity force). It is therefore time to study collective oscillations modes in lattices composed of such types of grains.

In the one-dimensional chain of spherical particles levitating in a plasma, the possible lattice oscillations modes are associated with the particle motion either in horizontal [7] or vertical [8,9] directions. As has been previously noticed [10], in the case of the rodlike particles, additional modes appear due to the new (rotational) degree of freedom. The "liquid crystal" lattices composed of rods will exhibit the rotational oscillation modes, similar to those in liquid crystals. Excitation and interactions of all these modes will lead to new types of phase transitions and affect those phase transitions existing also in lattices composed of spherical grains. In this Rapid Communication, we report the results of the first attack on the problem of oscillations in the lattice consisting of rod-shaped particles.

The simplest case corresponds to the rods with given (and static) charge distribution [10] along the rod length. More complicated is the case when the interaction of the rodlike particles among themselves and with the plasma is studied dynamically together with their charging, thus demanding that the problem of the charging of rods by the surrounding plasma should first be solved. At present, there are only first attempts [6] to tackle the latter. Therefore, some simplifying assumptions should be made in order to proceed with the calculation of the oscillation characteristics. Here, we model the rodlike particle with the rotator having two charges (and masses) concentrated on the ends of the rod. For further simplicity, we assume that the charges are fixed and the masses are equal. The rod of the length L, connecting these two charges, has zero radius and mass.

Consider the geometry sketched in Fig. 1. The one-dimensional rod chain is along the x axis, with the distance d between the centers of masses of the (unperturbed) rotators, $\mathbf{R}^n$ is the radius vector of the center of mass of the nth rotator (in our case of equal masses m at the rod ends, the center of mass is located in the center of the rotator, at the distance $L/2$ from its ends), the angle Θ^n is between the nth rotator and the z axis, and the angle ϕ^n is between the x axis and the projection of the nth rotator on the xy plane. Assume that, at the upper end of the nth rod, there is a pointlike particle (coordinate $\mathbf{a}^n$) with the charge q_a and mass m_a, and at the lower end of the same rod there is another pointlike particle (coordinate $\mathbf{b}^n$) with the charge q_b and mass m_b. Furthermore, we assume that the masses of the particles are equal, $m_a = m_b = m$, and the corresponding generalization to the case of unequal masses is trivial, with the respective change of the position of the center of mass along the rotator.

The radius vectors of the nth rod ends are $\mathbf{a}^n = nd\mathbf{e}_x + \mathbf{R}^n + L\mathbf{S}^n/2$ and $\mathbf{b}^n = nd\mathbf{e}_x + \mathbf{R}^n - L\mathbf{S}^n/2$, where $\mathbf{S}^n = (\cos\phi^n \sin\Theta^n, \sin\phi^n \sin\Theta^n, \cos\Theta^n)$. For the four distances between the ends of the nth and $(n+1)$th rotators we thus have

$$\mathbf{r}^{n+}_{a(b)a} = \mathbf{a}^n - \mathbf{a}(\mathbf{b})^{n+1} = -d\mathbf{e}_x + (\mathbf{R}^n - \mathbf{R}^{n+1}) + L(\mathbf{S}^n \mp \mathbf{S}^{n+1})/2,$$

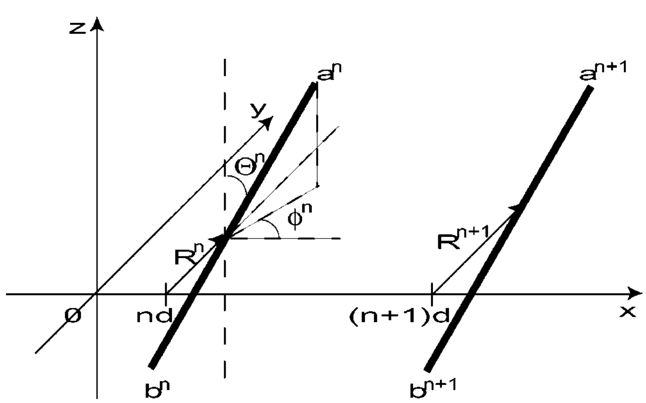

FIG. 1. Sketch of the considered geometry. The rotators of the length L separated at the distance d are assumed to be at the angles Θ and ϕ with respect to the reference frame.

*Email address: S.Vladimirov@physics.usyd.edu.au; URL: http://www.physics.usyd.edu.au/~vladimi

$$\mathbf{r}_{b(a)b}^{n+}=\mathbf{b}^n-\mathbf{a}(\mathbf{b})^{n+1}=-d\mathbf{e}_x+(\mathbf{R}^n-\mathbf{R}^{n+1})-L(\mathbf{S}^n\mp\mathbf{S}^{n+1})/2. \quad (1)$$

Here, the upper sign on the right-hand side corresponds to $\mathbf{r}_{aa}$ or $\mathbf{r}_{bb}$, and the lower sign on the right-hand side corresponds to $\mathbf{r}_{ba}$ or $\mathbf{r}_{ab}$, respectively. For the distances between the $(n-1)$th rod and the nth rod (e.g., for $\mathbf{r}_{aa}^{n-}$), we have similar expressions with the simultaneous change $(n+1)\to(n-1)$ and $d\to -d$.

Accounting for the oscillatory and rotational degrees of freedom and assuming the nearest-neighbor interactions, the Lagrangian [11] of the system is written as

$$\mathcal{L}=\frac{m}{2}\sum_n (\dot{\mathbf{R}}^n)^2+\frac{I_0}{2}\sum_n [(\dot{\phi}^n)^2\sin^2\Theta^n+(\dot{\Theta}^n)^2] -q_a\sum_n \Phi_{\text{ext}}(\mathbf{a}^n)-q_b\sum_n \Phi_{\text{ext}}(\mathbf{b}^n)-q_a\sum_n [\Phi_a(\mathbf{r}_{aa}^{n+}) +\Phi_b(\mathbf{r}_{ba}^{n+})]-q_b\sum_n [\Phi_a(\mathbf{r}_{ab}^{n+})+\Phi_b(\mathbf{r}_{bb}^{n+})] -q_a\sum_n [\Phi_a(\mathbf{r}_{aa}^{n-})+\Phi_b(\mathbf{r}_{ba}^{n-})] -q_b\sum_n [\Phi_a(\mathbf{r}_{ab}^{n-})+\Phi_b(\mathbf{r}_{bb}^{n-})], \quad (2)$$

where $I_0=mL^2/2$ is the moment of inertia of the considered rotator, $\Phi_a[\mathbf{r}_{aa}^{n+}]$ is the interaction potential between the nth and $(n+1)$th a particles, etc., and $\Phi_{\text{ext}}(\mathbf{r})$ is the external potential. Furthermore, we assume for the interaction potential the Debye approximation $\Phi_a(\mathbf{r})=(q_a/|\mathbf{r}|)\exp(-|\mathbf{r}|/\lambda_D)$, where λ_D is the plasma Debye length; the external potential is determined by the action of the gravity and the sheath electric field in the point of levitation; both fields act only along the z axis.

The Lagrangian equations of motion are given by

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{s}}-\frac{\partial\mathcal{L}}{\partial s}=0, \quad (3)$$

where $s=(\mathbf{R}^i,\Theta^i,\phi^i)$. General expressions for motion and rotation in all three dimensions are cumbersome and therefore, for simplicity, we assume that motions are only in the (x,z) plane such that $\mathbf{S}^n=(S_x^n,0,S_z^n)=(\sin\Theta^n,0,\cos\Theta^n)$, $\phi^n=0$, and $\mathbf{R}^n=(x^n,0,z^n)$. Thus we find for the motion of the center of mass in the x direction

$$2m\ddot{\mathbf{R}}^n=-q_a\left[\frac{\Phi'(\mathbf{r}_{aa}^{n+})}{|\mathbf{r}_{aa}^{n+}|}\mathbf{r}_{aa}^{n+}+\frac{\Phi'(\mathbf{r}_{aa}^{n-})}{|\mathbf{r}_{aa}^{n-}|}\mathbf{r}_{aa}^{n-}+\frac{\Phi'(\mathbf{r}_{ba}^{n+})}{|\mathbf{r}_{ba}^{n+}|}\mathbf{r}_{ba}^{n+} +\frac{\Phi'(\mathbf{r}_{ba}^{n-})}{|\mathbf{r}_{ba}^{n-}|}\mathbf{r}_{ba}^{n-}+\frac{\partial\Phi_{\text{ext}}(\mathbf{a}^n)}{\partial\mathbf{R}^n}\right]-q_b[\cdots], \quad (4)$$

where the dots stay for terms analogous to those in the first square brackets (with the interchange $a\leftrightarrow b$), and $\Phi_a'(\mathbf{r})\equiv d\Phi_a(\mathbf{r})/d|\mathbf{r}|=-(q_a/|\mathbf{r}|)(\lambda_D^{-1}+|\mathbf{r}|^{-1})\exp(-|\mathbf{r}|/\lambda_D)$ for the Debye interaction potential.

Rotations on the angle Θ are described by

$$I_0\ddot{\Theta}^n=-q_aL\left\{\frac{\Phi'(\mathbf{r}_{aa}^{n+})}{|\mathbf{r}_{aa}^{n+}|}[r_{aa,x}^{n+}\cos\Theta^n-r_{aa,z}^{n+}\sin\Theta^n] +\frac{\Phi'(\mathbf{r}_{aa}^{n-})}{|\mathbf{r}_{aa}^{n-}|}[r_{aa,x}^{n-}\cos\Theta^n-r_{aa,z}^{n-}\sin\Theta^n] +\frac{\Phi'(\mathbf{r}_{ba}^{n+})}{|\mathbf{r}_{ba}^{n+}|}[r_{ba,x}^{n+}\cos\Theta^n-r_{ba,z}^{n+}\sin\Theta^n] +\frac{\Phi'(\mathbf{r}_{ba}^{n-})}{|\mathbf{r}_{ba}^{n-}|}[r_{ba,x}^{n-}\cos\Theta^n-r_{ba,z}^{n-}\sin\Theta^n]+\frac{\partial\Phi_{\text{ext}}(\mathbf{a}^n)}{\partial\Theta^n}\right\} -q_bL\{\cdots\}. \quad (5)$$

General equations (4) and (5) can be used not only to obtain dispersion relations for small amplitude oscillations, but also to study mode interactions for larger amplitudes. For small deviations from the equilibrium, the oscillations decouple, and linear dispersion relations can be derived.

According to the experiments [6], there are two preferred equilibrium positions of levitating rodlike particles: when the rotators are oriented vertically (i.e., along the z axis in our geometry, see Fig. 1) and when the rotators are oriented horizontally (i.e., along the x axis in our geometry). Thus, below we consider dispersion relations for modes associated with small deviations around these two equilibrium positions.

If there are only small oscillations $\delta x\ll\min(d,L)$ of the centers of mass of vertically oriented $(\Theta=0)$ rotators in the x direction (which is horizontal, see Fig. 1), we find from Eq. (4)

$$2m\,\delta\ddot{x}^n=-\left\{q_a\left[\Phi_a''(d)+\frac{\Phi_b'(L_d)}{L_d}+\frac{d^2}{L_d}\left[\frac{\Phi_b'(r)}{r}\right]'_{L_d}\right] +q_b[\cdots]\right\}(2\,\delta x^n-\delta x^{n+1}-\delta x^{n-1}), \quad (6)$$

where $L_d=(d^2+L^2)^{1/2}$. For perturbations propagating along the x axis, Eq. (5) gives the following dispersion equation of the lattice-acoustic type:

$$\omega^2=\frac{4}{2m}\left[q_a\Phi_a''(d)+q_b\Phi_b''(d)+2q_a\frac{d^2}{L_d^2}\Phi_b''(L_d) +2q_a\frac{L^2}{L_d^3}\Phi_b'(L_d)\right]\sin^2(kd/2). \quad (7)$$

Here, we have taken into account that for the Debye interaction potential $q_a\Phi_b=q_b\Phi_a$. In the limit $L\ll d$ we recover from (7) the standard dispersion relation for the lattice-acoustic wave [7] in the chain of particles with the charge q_a+q_b and the mass $2m$:

$$\omega^2=4\frac{q_a+q_b}{2m}(\Phi_a''(d)+\Phi_b''(d))\sin^2(kd/2), \quad (8)$$

where $\Phi_a''(d)+\Phi_b''(d)=(q_a+q_b)(2+2d/\lambda_D+d^2/\lambda_D^2)\exp(-d/\lambda_D)/d^3$ for the interaction potential of Debye type.

For the horizontally oriented rod chain ($\Theta=\pi/2$, note that in this case we obviously have $d>L$) we find

$$2m\,\delta\ddot{x}^n=-[q_a\Phi_a''(d)+q_b\Phi_b''(d)](2\,\delta x^n-\delta x^{n+1}-\delta x^{n-1}) -[q_a\Phi_b''(d-L)+q_b\Phi_a''(d+L)](\delta x^n-\delta x^{n+1}) -[q_a\Phi_b''(d+L)+q_b\Phi_a''(d-L)](\delta x^n-\delta x^{n-1}). \tag{9}$$

From Eq. (9) (noting that in the case of the Debye interaction potential $q_a\Phi_b=q_b\Phi_a$) we obtain the dispersion relation for the acoustic mode

$$\omega^2=\frac{4}{2m}[q_a\Phi_a''(d)+q_b\Phi_b''(d)+q_a\Phi_b''(d-L)+q_b\Phi_a''(d+L)]\sin^2(kd/2). \tag{10}$$

In the limit $L\ll d$ from Eq. (10) we again recover Eq. (8).

For small oscillations in the vertical direction of the vertically oriented rotators (i.e., parallel to the z axis) we obtain

$$2m\,\delta\ddot{z}^n=-q_a\left\{\left[\frac{\Phi_a'(d)}{d}+\frac{\Phi_b'(L_d)}{L_d}+\frac{L^2}{L_d}\left[\frac{\Phi_b'(r)}{r}\right]'_{L_d}\right](2z^n-z^{n+1}-z^{n-1})+\frac{\partial\Phi_{\text{ext}}(\mathbf{a}^n)}{\partial z^n}\right\}-q_b\{\cdots\}. \tag{11}$$

For the parabolic approximation of the external potential (depending only on z) $q_a\Phi_{\text{ext}}(\mathbf{a}^n)+q_b\Phi_{\text{ext}}(\mathbf{b}^n)=\gamma_v(z^n-L/2)^2/2+\gamma_v(z^n+L/2)^2/2$, where $\gamma_{a,b}>0$, we obtain for the wave propagating along the chain the dispersion relation of the optical character [8]

$$\omega^2=\frac{\gamma_v}{m}+\frac{4}{2m}\left\{q_a\left[\frac{\Phi_a'(d)}{d}+\frac{\Phi_b'(L_d)}{L_d}+\frac{L^2}{L_d}\left[\frac{\Phi_b'(r)}{r}\right]'_{L_d}\right]+q_b[\cdots]\right\}\sin^2(kd/2). \tag{12}$$

In the limit $L\ll d$ we recover from Eq. (12) the dispersion relation of the optical-like mode [8] propagating in the chain of particles

$$\omega^2=\frac{\gamma_v}{m}+4\frac{q_a+q_b}{2m}\left[\frac{\Phi_a'(d)}{d}+\frac{\Phi_b'(d)}{d}\right]\sin^2(kd/2), \tag{13}$$

where $\Phi_a'(d)/d+\Phi_b'(d)/d=-(q_a+q_b)(1+d/\lambda_D)\exp(-d/\lambda_D)/d^3$ for the interaction potential of Debye type.

For vertical oscillations of horizontally oriented rotators we have (note that $d>L$)

$$2m\,\delta\ddot{z}^n=-\left[q_a\frac{\Phi_a'(d)}{d}+q_b\frac{\Phi_b'(d)}{d}\right](2\,\delta z^n-\delta z^{n+1}-\delta z^{n-1}) -\left[q_a\frac{\Phi_b'(d-L)}{d-L}+q_b\frac{\Phi_a'(d+L)}{d+L}\right](\delta z^n-\delta z^{n+1}) -\left[q_a\frac{\Phi_b'(d+L)}{d+L}+q_b\frac{\Phi_a'(d-L)}{d-L}\right](\delta z^n-\delta z^{n-1}) -q_a\frac{\partial\Phi_{\text{ext}}(\mathbf{a}^n)}{\partial z^n}-q_b\frac{\partial\Phi_{\text{ext}}(\mathbf{b}^n)}{\partial z^n}. \tag{14}$$

For the parabolic external potential (note that in this case we assume $q_a\Phi_{\text{ext}}(\mathbf{a}^n)+q_b\Phi_{\text{ext}}(\mathbf{b}^n)=\gamma_h(z^n)^2$) and the Debye interaction potential ($q_a\Phi_b=q_b\Phi_a$) we obtain from Eq. (14) the dispersion relation for the optic mode

$$\omega^2=\frac{\gamma_h}{m}+\frac{4}{2m}\left[q_a\frac{\Phi_a'(d)}{d}+q_b\frac{\Phi_b'(d)}{d}+q_a\frac{\Phi_b'(d-L)}{d-L}+q_b\frac{\Phi_a'(d+L)}{d+L}\right]\sin^2(kd/2). \tag{15}$$

In the limit $L\ll d$ from Eq. (15) we recover an equation similar to (13). Note that we assumed a slightly different character of the external potential to allow for the cases of stable vertically or, respectively, horizontally oriented rotators.

Small rotating oscillations around $\Theta=0$ (i.e., for vertically oriented rotators) are described by

$$\frac{2I_0}{L^2}\delta\ddot{\Theta}^n=-q_a\left\{\Phi_a''(d)(2\Theta^n-\Theta^{n+1}-\Theta^{n-1})+\left[\frac{\Phi_b'(L_d)}{L_d}+\frac{d^2}{L_d}\left[\frac{\Phi_b'(r)}{r}\right]'_{L_d}\right](2\Theta^n+\Theta^{n+1}+\Theta^{n-1})+\frac{2}{L}\frac{\partial\Phi_{\text{ext}}(\mathbf{a}^n)}{\partial\Theta^n}\right\}-q_b\{\cdots\}. \tag{16}$$

Here, we note the changed character of the dispersion related to the nearest-neighbor interactions, as compared with the previous cases of oscillations of the rotators' centers of mass (see terms with $2\Theta^n+\Theta^{n+1}+\Theta^{n-1}$). In the case of the external parabolic potential supporting the vertical orientation of the rotators (and accounting for $q_a\Phi_b=q_b\Phi_a$), we obtain the following dispersion:

$$\omega^2=\frac{2\gamma_v}{m}+\frac{8}{mL_d^3}[q_a d^2 L_d\Phi_b''(L_d)+q_a L^2\Phi_b'(L_d)]+\frac{4}{m}\left[q_a\Phi_a''(d)+q_b\Phi_b''(d)-2q_a\frac{d^2}{L_d^2}\Phi_b''(L_d)-2q_a\frac{L^2}{L_d^3}\Phi_b'(L_d)\right]\sin^2(kd/2). \tag{17}$$

Note that for the Debye-type interaction potential, the factor at the oscillating term on the right-hand side (rhs) of Eq. (17)

is always positive. This means that although in general there is the frequency gap, similar (but not equal) to that of the vertical oscillations of the center of mass of the rotators, the dispersion character is different compared with the case of the vertical vibrations: Because of the sign of the factor in front of the dispersion term in (17), there is no anomalous dispersion (i.e., the mode frequency increases with the increase of the wave number). We also note the second term on the rhs of (17): if positive, it allows rotational oscillations (in xz plane) in the chain of vertically oriented rotators even in the absence of an external confining (in z) potential due to the interactions with the nearest neighbors. For the Debye screening potential, this term can be either positive or negative depending on the relations between L, d, and the plasma Debye length λ_{De}. This property indicates that the system can be unstable with respect to rotations of rods on the angle Θ; this can have important consequences on the excitation of the corresponding mode and related phase transition associated with the rotational (in)stability in the chains of rotators. Indeed, by changing the plasma characteristics, the originally stable vertically oriented equilibrium state of rotators can change its character and become unstable (and vice versa). Moreover, for marginally unstable equilibrium, because of the normal character of the wave dispersion, for some wavelengths the oscillations still can be stable (i.e., when the positive dispersive term exceeds the negative nondispersive term).

Finally, consider rotational oscillations of horizontally oriented rotators (around $\Theta=\pi/2$). We assume $\Theta=\pi/2-\vartheta$ and obtain [compare with Eq. (14)]

$$\begin{aligned}\frac{2I_0}{L^2}\delta\ddot{\vartheta}^n=&-\left[q_a\frac{\Phi_a'(d)}{d}+q_b\frac{\Phi_a'(d)}{d}\right](2\delta\vartheta^n-\delta\vartheta^{n+1}-\delta\vartheta^{n-1})\\&+\left[q_a\frac{\Phi_b'(d-L)}{d-L}+q_b\frac{\Phi_a'(d+L)}{d+L}\right](\delta\vartheta^n-\delta\vartheta^{n+1})\\&+\left[q_a\frac{\Phi_b'(d+L)}{d+L}+q_b\frac{\Phi_a'(d-L)}{d-L}\right](\delta\vartheta^n-\delta\vartheta^{n-1})\\&-\frac{d}{L}\left[q_a\frac{\Phi_b'(d-L)}{d-L}+q_b\frac{\Phi_a'(d-L)}{d-L}-q_a\frac{\Phi_b'(d+L)}{d+L}\right.\\&\left.-q_b\frac{\Phi_a'(d-L)}{d-L}\right]\delta\vartheta^n-\frac{2q_a}{L}\frac{\partial\Phi_{\rm ext}(\mathbf{a}^n)}{\partial\vartheta^n}-\frac{2q_b}{L}\frac{\partial\Phi_{\rm ext}(\mathbf{b}^n)}{\partial\vartheta^n}.\end{aligned}\tag{18}$$

From the external potential supporting the horizontal orientation of the rotators (and, as usual, taking into account that for the Debye interaction potential $q_a\Phi_b=q_b\Phi_a$), we have

$$\begin{aligned}\omega^2=&\frac{2\gamma_h}{m}+\frac{4}{mL}[q_a\Phi_b'(d-L)-q_a\Phi_b'(d+L)]+\frac{4}{m}\left[q_a\frac{\Phi_a'(d)}{d}\right.\\&\left.+q_b\frac{\Phi_b'(d)}{d}+q_a\frac{\Phi_b'(d-L)}{d-L}+q_a\frac{\Phi_b'(d+L)}{d+L}\right]\sin^2(kd/2).\end{aligned}\tag{19}$$

Again, we note the second term on the rhs of Eq. (19) (originating from the nearest-neighbor interactions): for the Debye interaction potential, it is always negative and can prevent the stable equilibrium of horizontally oriented rotators even in the case of confining (in z) potential. Since the vertical confinement is usually associated with the properties of the plasma sheath (where the particles are levitated) [9], we see that again the change of plasma parameters can lead to the change of the stability characteristics of the horizontally oriented rods. Depending on the particular character of the interaction (and on the plasma parameters), this feature can affect excitation of the rotation modes of horizontally oriented rotators and therefore the related phase transitions. In the limit $L\to 0$ we recover dispersion (13) from Eq. (19).

To conclude, we studied oscillations in the chain of rotators interacting via the Debye type of potential and levitating in an external potential in a plasma. We demonstrated that new modes associated with the rotational degrees of freedom can propagate in such a system. New features associated with the rotational modes include interesting interplay of interactions of the rotators with a plasma (formalized above by terms containing the external potential) and with themselves. Combination of these interactions strongly affect the equilibrium positions and orientations of the rotators and therefore will influence phase transitions associated with such rotating modes. Further analysis can include computation of the rotators' levitation for particular plasma sheath models. General equations derived here also allow further analysis of mode interactions when perturbations in rotations together with perturbations in rotators' positions take place simultaneously. Some of these analyses are underway and will be reported elsewhere.

This work was supported by the Australian Research Council.

[1] H.M. Thomas *et al.*, Phys. Rev. Lett. **73**, 652 (1994).
[2] A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).
[3] J.H. Chu and Lin I, Physica A **205**, 183 (1994).
[4] H.M. Thomas and G.E. Morfill, Nature (London) **379**, 806 (1996).
[5] U. Mohideen, H.U. Rahman, M.A. Smith, M. Rosenberg, and D.A. Mendis, Phys. Rev. Lett. **81**, 349 (1998).
[6] B.M. Annaratone *et al.*, Phys. Rev. E **63**, 036406 (2001).
[7] F. Melandsø, Phys. Plasmas **3**, 3890 (1996).
[8] S.V. Vladimirov, P.V. Shevchenko, and N.F. Cramer, Phys. Rev. E **56**, 74 (1997).
[9] S.V. Vladimirov, N.F. Cramer, and P.V. Shevchenko, Phys. Rev. E **60**, 7369 (1999).
[10] S.V. Vladimirov and M. Nambu, Phys. Rev. E **64**, 026403 (2001).
[11] L.D. Landau and E.M. Lifshitz, *Mechanics* (Pergamon, Oxford, 1969).